CALCULATION OF THE EFFECTIVENESS OF PLATE COOLING WITH COOLANT EJECTION AT AN ANGLE TO THE MAIN FLOW WITH ALLOWANCE FOR THE STAGNANT ZONE

N. N. Kortikov and Yu. A. Smirnov

UDC 536.244:532.522

A method is presented for calculating the efficiency of cooling of a flat surface with large ejection angles. The gaseous wall screen is calculated from formulas for tangential ejection.

The technique of boundary cooling is one of the most effective methods of reducing the temperature of the blades of gas turbines. Effective methods have been developed [1, 2] for calculating the efficiency of boundary cooling with tangential coolant ejection onto the surface to be cooled. When ejection takes place at an angle to the surface, it is necessary to use empirical corrections [3]. A plane jet discharged at an angle to an entraining flow parallel to the surface may become reattached to it some distance from the ejection site, with the formation of a stagnant zone. The flow behind the reattachment point will be equivalent to the case of tangential ejection. The studies [4, 5] developed methods of calculating the dimensions of the stagnant zone formed under a jet discharged at an angle to a surface in a space filled with a stationary fluid. These methods give good agreement with experimental data in the absence of an entraining flow. In [3] the equations of equilibrium of the pressure, inertial, and centrifugal forces acting on an element of a jet were used to derive the equation of the jet axis for a jet ejected into an entraining flow. However, the results obtained here did not agree with the experimental data in [6] for ejection parameters greater than unity.

The goal of the present article is to determine the site of reattachment of a jet and its dynamic characteristics at this point within a broad range of ejection parameters. We also want to develop a method of calculating the gaseous wall screen in the case of large ejection angles (greater than 45°).

Determination of the Reattachment Region. Figure 1 shows the flow pattern with reattachment of the jet to the wall. A stagnant zone with fluid circulating in it is formed after the ejection site. For a plane jet of an incompressible fluid, the steady-state equations of motion, continuity, and equilibrium of the centrifugal and surface pressure forces, written in the system of coordinates x, y, have the form [4, 7]:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \tau}{\partial y},\quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$p_1+\frac{1}{2}C_D\rho u_1\sin^2\alpha=p_B+J/R \tag{2}$$

with the following conditions prevailing on the axis and boundaries of the jet:

$$u=u_1 \text{ at } y=-b;\ u=u_m \text{ at } y=0;\ u=0 \text{ at } y=b. \tag{3}$$

In writing Eqs. (1), (2), we assume that the transverse pressure gradient is significantly greater than the longitudinal gradient in the attachment region ($0 < x' < x_g'$), that the curvature of the streamlines is explicitly considered only in the expression for turbulent friction τ, and that the projection of the motion equation in the direction of the y axis is written in integral form.

To determine the rarefaction in the stagnant zone, we take the pressure scale in the form of the difference in pressures in the mouth of the jet and in the entraining flow. Determining the excess pressure in the jet at the ejection site by means of the Bernoulli integral and determining the pressure of the entraining flow as the stagnation pressure in its interaction with the jet, we obtain

M. I. Kalinin Leningrad Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 5, pp. 715-719, May, 1985. Original article submitted March 20, 1984.

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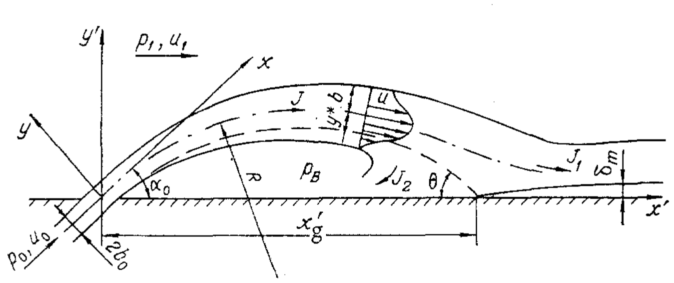

Fig. 1

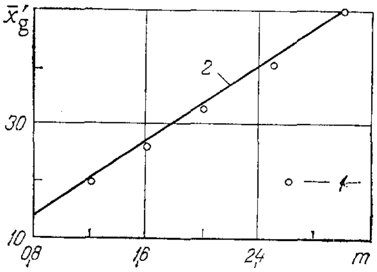

Fig. 2

Fig. 1. Flow pattern with reattachment of the jet to the wall.

Fig. 2. Dependence of the length of the stagnant zone $\bar{x}'_g$ on the ejection parameter m: 1) experiment [6]; 2) calculation with C_D = 2.6; α_0 = 60°.

$$p_0 - p_1 = \frac{\rho u_0^2}{2}(1 + \sin^2\alpha_0/m^2). \tag{4}$$

Experimental data shows that recirculation of the fluid in the stagnation zone results in a reduction in pressure compared to the pressure in the main flow [5]. Here, the amount of rarefaction is affected by the degree of suction of fluid from this zone into the jet, which in turn depends only on the relative difference in velocities in the stagnant zone and in the jet. This allows us to assume that the degree of rarefaction in the zone under the jet compared to the excess pressure in the mouth of the jet depends only slightly on the velocity of the entraining flow. This is expressed by the equation

$$(p_B - p_0)/(p_0 - p_1) = \text{idem}. \tag{5}$$

The expression for the dimensionless radius of curvature ot the jet axis ($\bar{R} = R/b_0$) is found by making Eq. (2) dimensionless. Here, Eq. (4) is used as the pressure scale. The momentum of the jet is the initial momentum J_0 and the length is half the width of the slit b_0:

$$\bar{R} = (2m^2 J/J_0)[-C_{pB}(m^2 + \sin^2\alpha_0) + C_D \sin^2\alpha]^{-1}, \tag{6}$$

where $C_{pB} = (p_B - p_1)/(p_1)/(p_0)$ is the pressure coefficient in the circulation zone in the absence of an entraining flow. This coefficient can be determined either by calculation or on the basis of the experimental data in [5]. The change in the dimensionless momentum $\bar{J} = J/J_0$ in the longitudinal direction is determined in (6) by the method of integral relations [8]. Here, Eq. (1) is replaced by an integral analog by integration over the thickness of the jet using boundary conditions (3):

$$\bar{J} = 1 - \frac{1}{2m}\left[1 - \int_0^{-b} (u/u_0)\, dy\right]. \tag{7}$$

There may be cases when the attachment point is within the limits of the initial or main sections. Thus, to determine the integral in (7), we use velocity profiles valid for the initial section of the jet

$$u = u_0 \text{ at } -\delta_1 < y < 0,\ (u - u_1)/(u_0 - u_1) = Q(\eta) \text{ at } -b < y < -\delta_1,$$
$$\eta = (y + \delta_1)/(-b + \delta_1)$$

and its main section

$$(u - u_1)/(u_m - u_1) = Q(\eta) \text{ at } -b < y < 0,\ \eta = -y/b.$$

Here, the function $Q(\eta) = (1 - \eta^{3/2})^2$ is the Schlichting profile. The flow separates into two parts at the attachment point: the portion of fluid with the momentum J_1 continues its motion along the surface; the portion with the momentum J_2 enters the circulation zone in the direction of the mouth of the jet (see Fig. 1). The boundary is the nominal dividing line with the coordinate y*. We determine J_2 from the law of momentum conservation at the jet attachment point

$$J_2 = J_1 - J g \cos\theta = \rho\int_{-b}^{0} u^2 dy - \rho\cos\theta \int_{-b}^{b} u^2 dy + \rho\int_0^{y^*} u^2 dy. \tag{8}$$

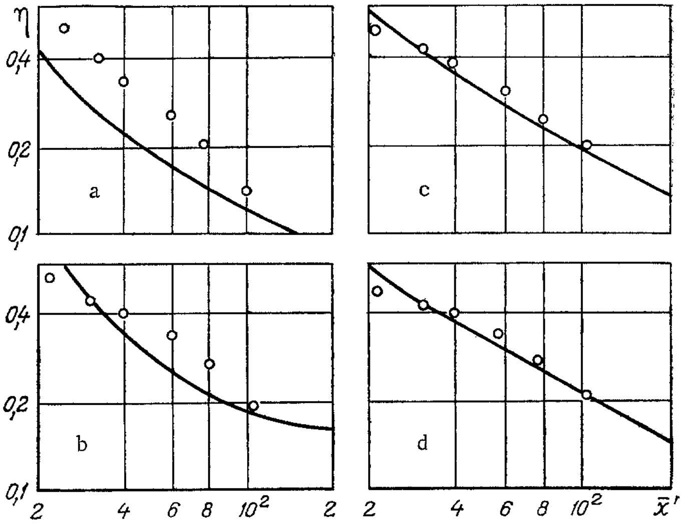

Fig. 3. Plate cooling efficiency with ejection of coolant at an angle to the main flow: 1) experiment [3]; 2) calculation, α_0 = 75°; a) m = 0.5; b) 0.8; c) 1.2; d) 1.5.

The last integral in the right-hand side of (8) is expressed through the momentum J_2. With allowance for this, we write the expressions for the momenta $\bar{J}_1$ and $\bar{J}_2$ in the form

$$\bar{J}_1 = J_1/J_0 = \frac{1}{2}(1+\cos\theta)\frac{J_g}{J_0}, \quad \bar{J}_2 = J_2/J_0 = \frac{1}{2}(1-\cos\theta)\frac{J_g}{J_0}. \tag{9}$$

Thus, Eqs. (6) and (9) make it possible, with a preassigned step, to calculate the curvature of the jet at all points of its path beginning from the mouth. This in turn makes it possible to determine the point of reattachment x_g' and the dimensionless momentum of the jet $\bar{J}_g$ at this point. The calculated results and their comparison with the available literature data in Fig. 2 show that there is adequate agreement in the ejection parameter range 0.8 < m < 3.

Cooling Efficiency. The flow behind the reattachment point can be regarded as the propagation of a turbulent wall jet in the case of tangential ejection with the momentum J_1 at $x' = x_g'$. The effectiveness of the gas screen under adiabatic conditions is assumed to be characterized by the dimensionless temperature of the wall $\eta = (T_1 - T_{ad})/(T_1 - T_0)$ [9]. In solving problems concerning the efficiency of a gas screen, one of the simplest and most convenient methods is that based on the use of integral equations for the boundary layer of a turbulent wall jet. Using the equation of conservation of excess enthalpy, we represent the expression for cooling efficiency in the form [10]

$$\eta_\infty = \left[\frac{12}{13}\bar{t}\,\bar{\delta}_m\left(1-\frac{13}{25}\frac{\bar{\delta}_m}{\bar{b}}\right)+\frac{9}{28}\bar{t}\frac{(\bar{b}-\bar{\delta}_m)^2}{\bar{b}}+\frac{5}{28}\frac{1}{m_t}\frac{(\bar{b}-\bar{\delta}_m)^2}{\bar{b}}\right]^{-1}, \tag{10}$$

where $\bar{t} = u_{mt}/u_{0t} = 3.6(x/b_0)^{-0.5}$ [2]; $\bar{\delta}_m = \delta_m/b_0$ is the dimensionless thickness of the boundary layer of the semiinfinite jet.

Equation (10) is valid at large distances from the ejection site, since it was derived with the use of relations that are valid for the main section of the wall jet ($x' \to \infty$). Following [1], we propose the interpolation formula $\eta = (1 + \eta_\infty^{\alpha})^{d}$ to expand the range of application of Eq. (10). This formula embraces the entire range of efficiency of boundary cooling.

Let us calculate the ejection parameter m_t for an equivalent wall jet which is different from the ejection parameter of the initial jet discharged at an angle to the flow. Ignoring friction on the section from the ejection site to the attachment point, we find that transformation of the momentum equation and use of the condition of equality of the momenta of the initial and equivalent jets yields the following quadratic equation in m_t:

$$m_t^2 + m_t\left(\frac{u_{mt}}{u_{0t}}\bar{b}\int_0^1 Q\,d\eta - 1\right) - 2m^2\bar{J}_1 = 0. \tag{11}$$

The width of the mixing zone $\bar{b}$ does not depend on m_t [9] in the range of ejection parameters realized in actual film cooling ($0.5 < m_t < 2.0$). This allows us to regard (11) as a quadratic equation with constant coefficients, where a positive root is chosen as the solution. Figure 3 shows results of calculation of cooling efficiency with ejection at an angle to the main flow. Also shown is experimental data from [3]. Satisfactory agreement with the experimental data is seen in the range $0.5 < m < 1.5$ at $a = -1$, $d = -0.8$. An increase in m is accompanied by an improvement in this agreement. Ejection of coolant at an angle adversely affects the protective properties of the jet and leads to a reduction in efficiency compared to tangential ejection.

Thus, the above physical model makes it possible to calculate the efficiency of a gas screen downstream from a stagnant zone with an accuracy which is sufficient for engineering purposes (error no larger than 20%).

NOTATION

x, y coordinates directed along a tangent and a normal to the jet axis; x', y', coordinates connected to the surface; u, v, longitudinal and transverse components of velocity in a jet with the coordinates x, y; C_D, coefficient of resistance of jet to the entraining flow; α, angle between the vectors of velocities u and u_1; θ, angle of incidence of jet at point of attachment; R, radius of curvature of jet axis; p_0, p_1, p_B, static pressures in the jet, main flow, and circulation zone; $J_0 = 2\rho b_0 u_0^2$, initial momentum of jet; y*, coordinate of the dividing line; $m = \rho_0 u_0/\rho_1 u_1$, ejection parameter; δ_1, boundary of potential core of jet within the initial section. Indices: 0, mouth of jet; 1, main flow; t, tangential wall jet.

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